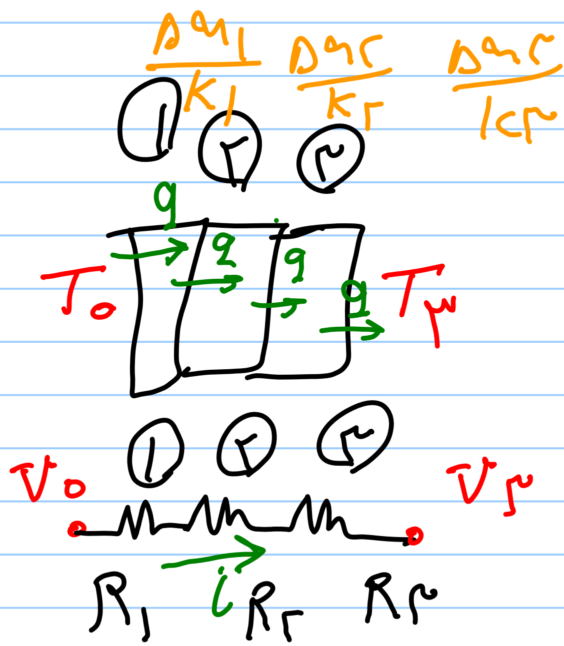
Note Title 6/1/2020

به نام خدا

جلسه یازدهم مبانی پدیده‌های انتقال

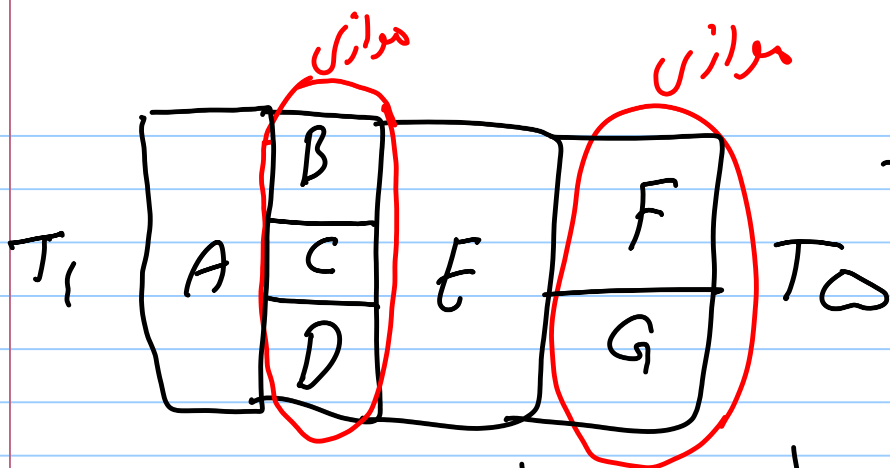
لایه‌ای ← مدار معادل الکتریکی (مقاومت)

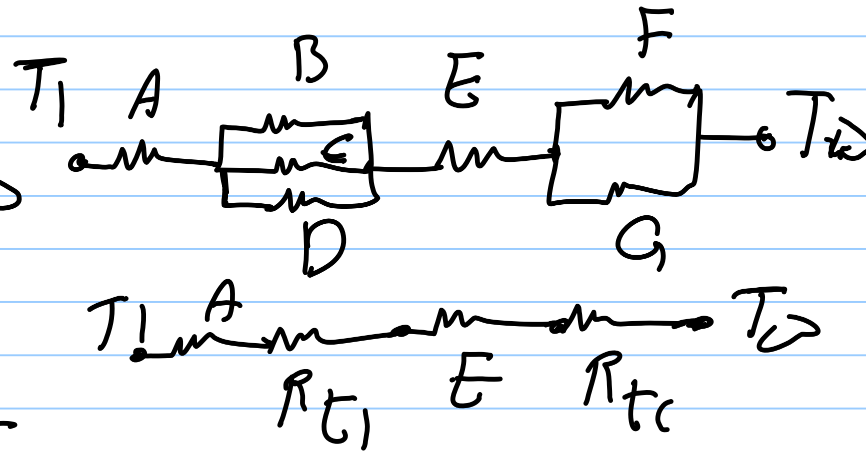


$$q = \frac{T_0 - T_3}{\frac{\Delta x_1}{k_1} + \frac{\Delta x_2}{k_2} + \frac{\Delta x_3}{k_3}}$$

$$I = \frac{V}{R}$$

$$R_t = R_1 + R_2 + R_3$$





$$\frac{1}{R_{t_1}} = \frac{1}{R_B} + \frac{1}{R_C} + \frac{1}{R_D}$$

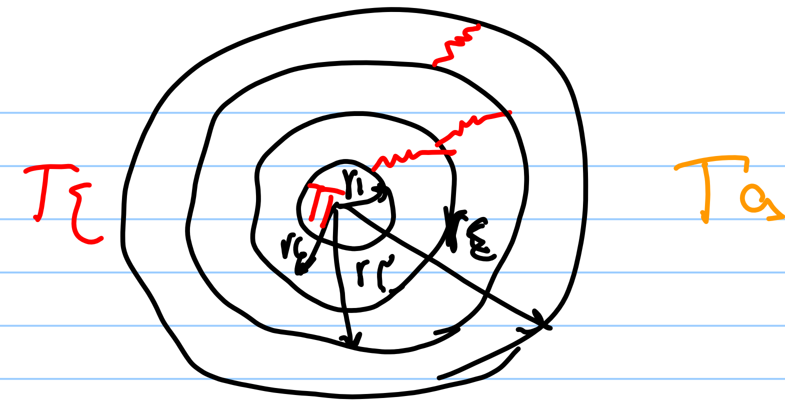
$$\frac{1}{R_{t_2}} = \frac{1}{R_F} + \frac{1}{R_G}$$

$$R_T = R_A + R_{t_1} + R_E + R_{t_2}$$

$$q = \frac{T_1 - T_\infty}{R_T}$$

$$q = \frac{(T_i - T_o)\,2\pi}{\dfrac{\ln \dfrac{R_o}{R_c}}{K}}$$

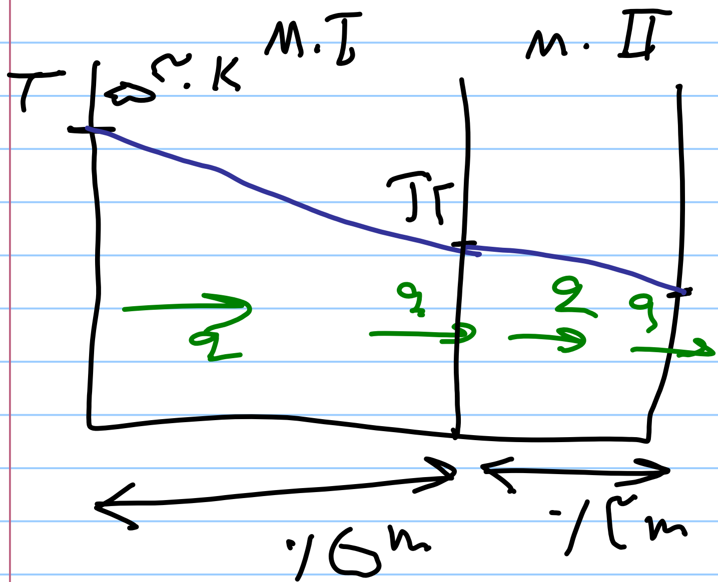
T_a



$$q = \frac{2\pi\,(T_1 - \cancel{T_\varepsilon}\ T_a)}{\dfrac{\ln \dfrac{r_c}{r_1}}{k_1} + \dfrac{\ln \dfrac{r_c'}{r_c}}{k_c} + \dfrac{\ln \dfrac{r_\varepsilon}{r_c'}}{k_p} + \dfrac{1}{r_\varepsilon h}}$$

$$\frac{\ln \dfrac{r_c}{r_1}}{k_1} \qquad \frac{\ln \dfrac{r_c'}{r_c}}{k_c} \qquad \frac{\ln \dfrac{r_\varepsilon}{r_c'}}{k_p}$$

مثال : نمودار درجه حرارت یک سیستم در شرایط تعادل به صورت زیر است. ضریب هدایت ماده II را در صورت داشتن شار حرارتی $12.06\times10^3\ W/m^2$ و ضریب هدایت ماده I $52\ W m^{-1}K^{-1}$ محاسبه نمایید.



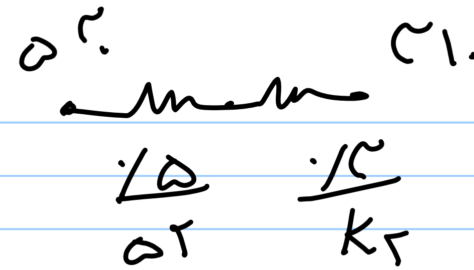
$$q = 12.06\times10^3\ W/m^2 = -52\frac{(T_2-500)}{0.5}$$

$$T_2 = 414K$$

$$q = 12.06\times10^3\ W/m^2 = -K_2\frac{(310-414)}{0.2}$$

$$K_2 = 34.17\ W m^{-1}K^{-1}$$

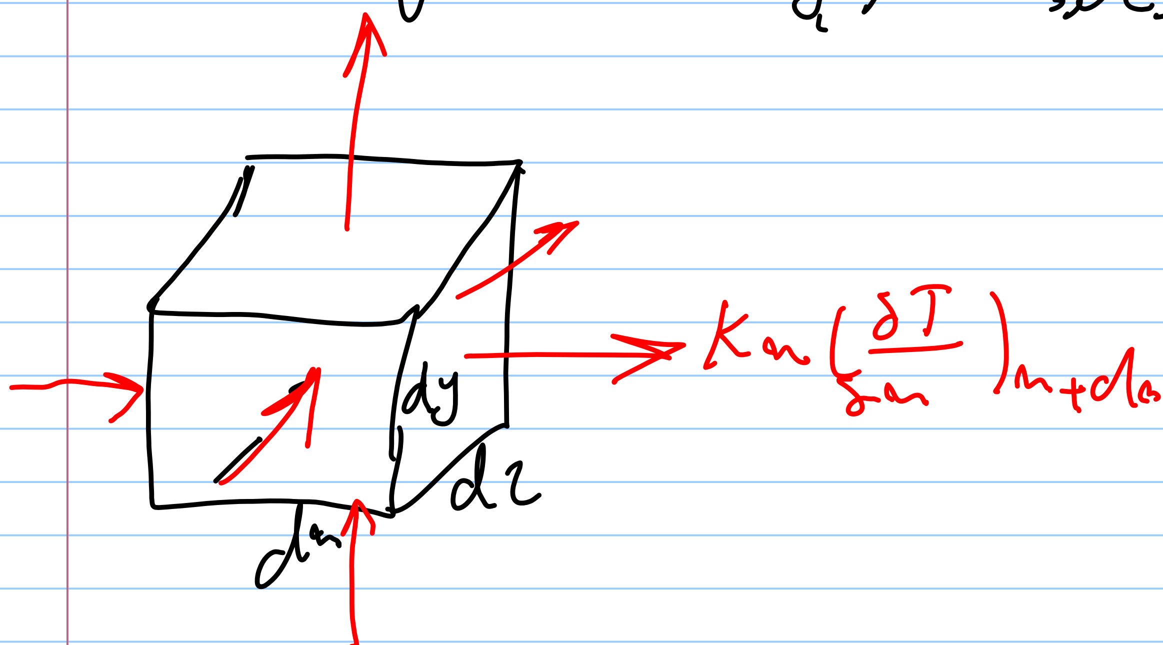
$$q = ١٢,٠٤٧١,٣ = \frac{٥٣,٠ - ٣١,٠}{\frac{٠/٥}{٥٢} + \frac{٠/٣}{k_٢}}$$



$$k_٢ = ٣٤,١٧$$

هدایت حرارتی ناپایدار

حرارت ذخیره شده = حرارت تولید شده + حرارت ورودی − حرارت خروجی



$$\left[-k_x\left(\frac{\partial T}{\partial x}\right)_x + k_x\left(\frac{\partial T}{\partial x}\right)_{x+dx}\right]dy\,dz + \left[-k_y\left(\frac{\partial T}{\partial y}\right)_y + k_y\left(\frac{\partial T}{\partial y}\right)_{y+dy}\right]dx\,dz + \left[-k_z\left(\frac{\partial T}{\partial z}\right)_z + k_z\left(\frac{\partial T}{\partial z}\right)_{z+dz}\right]dx\,dy$$

$$+\ \dot{q}(x,y,z)\,dx\,dy\,dz = \frac{\partial}{\partial t}(\rho c_p T)\,dx\,dy\,dz$$

بسط تیلور $\quad k_x\left(\frac{\partial T}{\partial x}\right)_{x+dx} = k_x\left(\frac{\partial T}{\partial x}\right)_x + \frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right)dx$

$$\Rightarrow \boxed{\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right) + \dot{q}(x,y,z) = \frac{\partial}{\partial t}\left(\rho c_p T\right)}$$

فرض کنیم ماده همسانگرد $k = k_x = k_y = k_z$

c_p و ρ تابع دما نباشند

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}(x,y,z)=\rho c_p\frac{\partial T}{\partial t}$$

$K = Cte$

$$\frac{K}{\rho c_p}\left[\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right]+\frac{\dot{q}(x,y,z)}{\rho c_p}=\cancel{\rho c_p}\frac{\partial T}{\partial t}$$

$\frac{K}{\rho c_p} \rightarrow \alpha$

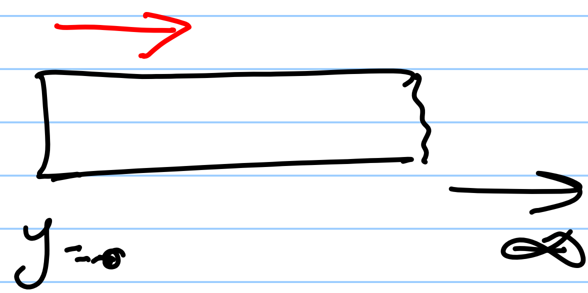
$\alpha = \frac{K}{\rho c_p} \rightarrow$ ضریب نفوذ حرارتی

در شرایط پایا $\frac{\partial T}{\partial t} = 0$ و در شرایط ناپایا $\frac{\partial T}{\partial t} \neq 0$

مختصات استوانه‌ای

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\dot{q}(r)}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

حل معادله حرارت در یک جسم نیمه بی‌نهایت :



$$\alpha \frac{\partial^2 T}{\partial y^2} = \frac{\partial T}{\partial t}$$

$$T(y,t)$$

$$T = A + B\,\mathrm{erfc}\left(\frac{y}{2\sqrt{\alpha t}}\right)$$

$$\operatorname{erf}(y) = \frac{۲}{\sqrt{\pi}} \int_{۰}^{y} e^{-y^{۲}}\, dy \qquad \operatorname{erfc}(y) = ۱ - \operatorname{erf}(y)$$

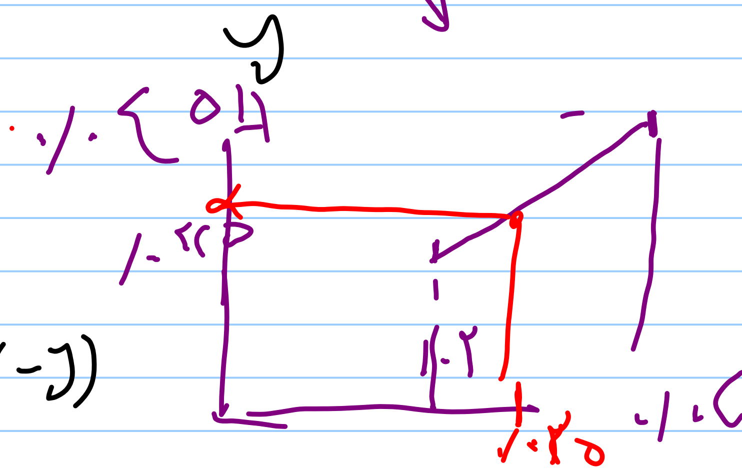
y	erf(y)
۰	۰
۰٫۲	۰٫۲۲۵
۰٫۲۵	
۰٫۴	۰٫۴۵۱۱

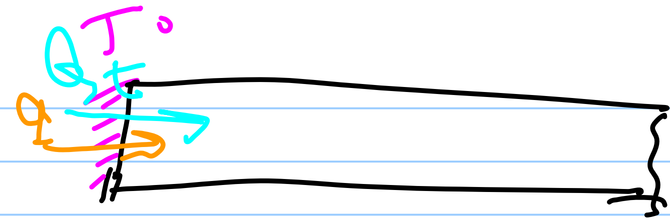
$$\frac{۰٫۴۵۱۱ - ۰٫۲۲۵}{۰٫۴ - ۰٫۲} = \frac{\operatorname{erf}(۰٫۲۵) - ۰٫۲۲۵}{۰٫۲۵ - ۰٫۲}$$

$\operatorname{erf}(۰) = ۰$

$\operatorname{erf}(\infty) = ۱$

$-\operatorname{erf}(y) = \operatorname{erf}(-y)$





$y = 0$ $\qquad$ $y = \infty$

$$\alpha \frac{\partial^2 T}{\partial y^2} = \frac{\partial T}{\partial t}$$

$T_i = \cancel{0}$ $\ T_i$

$$T = A + B\,\mathrm{erfc}\left(\frac{y}{2\sqrt{\alpha t}}\right)$$

$T = 0 = T_i \quad t = 0 \quad y > 0$

$T = T_0 \quad t > 0 \quad y = 0$

✓ $T \to T_i = 0 \quad t > 0 \quad y \to \infty$

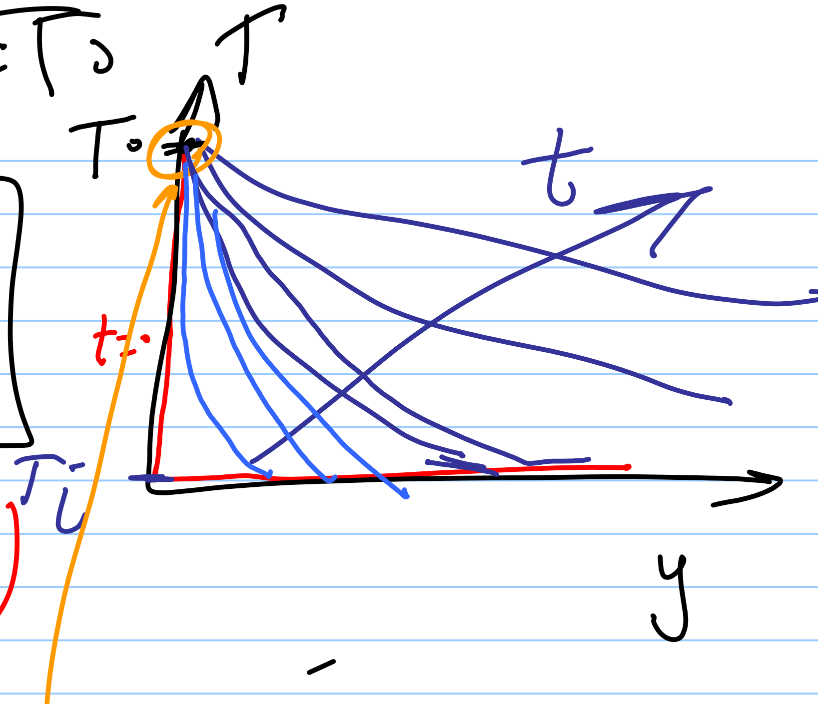
$$T = 0 = A + B(0) \Rightarrow A = 0$$

$$T = B\,\mathrm{erfc}\left(\frac{y}{2\sqrt{\alpha t}}\right)$$

$$T = T_0 = B(1) \Rightarrow B = T_0$$

$$\boxed{T = T_0 \, \mathrm{erfc}\left(\frac{y}{2\sqrt{\alpha t}}\right)}$$

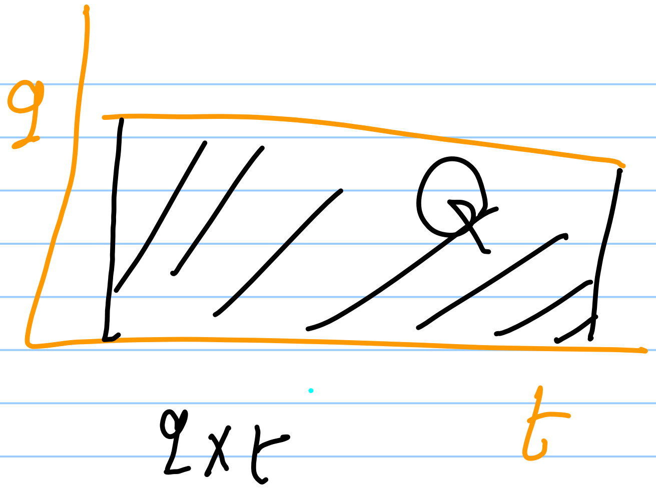
$$T - T_i = (T_0 - T_i)\, \mathrm{erfc}\left(\frac{y}{2\sqrt{\alpha t}}\right)$$

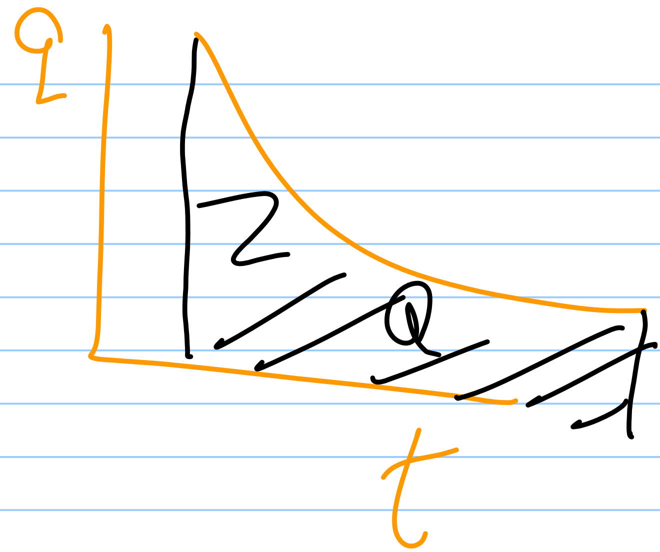


$$q_y = -k\left(\frac{\partial T}{\partial y}\right)$$

$$q_y = +k \frac{T_0 \, e^{-y^2/4\alpha t}}{\sqrt{\pi \alpha t}}$$

$y = 0$ $$q_{y=0} = \frac{k T_0}{\sqrt{\pi \alpha t}}$$





$Q =$, c_i
c_{t_i}

$$Q = \int_{0}^{t_e} \frac{k T_0}{\sqrt{\pi \alpha t}} dt = 2 k T_0 \left(\frac{t_e}{\pi \alpha} \right)^{1/2}$$

$$2k (T_0 - T_i) \left(\frac{t_e}{\pi \alpha} \right)^{1/2}$$